# First Grade

## 2018-2019 Guide

January 22nd -February 8th

## Eureka

# Module 3: *Ordering and Comparing Length Measurements as Numbers*

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

# Table of Contents

# Module 3 Performance Overview

- Module 3 opens in Topic A by extending students' Kindergarten experiences with direct length comparison to the new learning of indirect comparison whereby the length of one object is used to compare the lengths of two other objects. Students use transitivity, or indirect comparison, to compare short distances within the classroom.

- Topic B takes *longer than* and *shorter than* to a new level of precision by introducing the idea of a length unit. Centimeter cubes are laid alongside the length of an object as students learn that the total number of cubes laid end to end with no gaps or overlaps represents the length of that object. The Geometric Measurement Progressions Document expresses the research indicating the importance of teaching standard units to Grade 1 students before non-standard units. Thus, Grade 1 students learn about the centimeter before exploring non-standard units of measurement in this module.

- Topic C explores the usefulness of measuring with similar units. Students measure the same objects from Topic B using two different non-standard units, toothpicks and small paper clips, simultaneously to measure one object and answer questions. They realize that using iterations of the same unit will yield consistent measurement results. Similarly, students explore what it means to use a different unit of measurement from their classmates. It becomes obvious to students that if we want to have discussions about the lengths of objects, we must measure with the same units. With new understanding of consistent measurement, Topic C closes with students solving compare with difference unknown problems. Students use standard units to answer such questions.

- Topic D closes the module as students represent and interpret data. They collect data about their classmates and sort that information into three categories. Using same-sized pictures on squares, students represent this sorted data so that it can be easily compared and described. Students interpret information presented in the graphs by first determining the number of data points in a given category. The module closes with students asking and answering varied questions about data sets (compare with difference unknown). Their work with units representing data points is an application of students' earlier work with length as they observe that each square can be lightly interpreted as a length unit, which helps them analyze the data.

| Module 3: Ordering and Comparing Length Measurements as Numbers |||
|---|---|---|
| **Pacing:** January 22nd -February 8th **14 Days** |||
| **Topic** | **Lesson** | **Student Lesson Objective/ Supportive Videos** |
| **Topic A:** Indirect Comparison in Length Measurement | Lesson 1 | Compare length directly and consider the importance of aligning endpoints https://www.youtube.com/watch?v=47MnZV6jo74&list=PLvolZqLMhJmk7mcZ7SxjWhYG1YlSZ4oYC |
| | Lesson 2 | Compare length using indirect comparison by finding objects longer than, shorter than, and equal in length to that of a string https://www.youtube.com/watch?v=n6h8qNl9K0s&list=PLvolZqLMhJmk7mcZ7SxjWhYG1YlSZ4oYC&index=2 |
| | Lesson 3 | Order three lengths using indirect comparison. https://www.youtube.com/watch?v=t_WQWtPLfHI&list=PLvolZqLMhJmk7mcZ7SxjWhYG1YlSZ4oYC&index=3 |
| **Topic B:** Standard Length Units | Lesson 4 | Express the length of an object using centimeter cubes as length units to measure with no gaps or overlaps. https://www.youtube.com/watch?v=AvoB3V0zwGA&list=PLvolZqLMhJmk7mcZ7SxjWhYG1YlSZ4oYC&index=4 |
| | Lesson 5 | Rename and measure with centimeter cubes, using their standard unit name of centimeters https://www.youtube.com/watch?v=eYKOcSGpXLI&list=PLvolZqLMhJmk7mcZ7SxjWhYG1YlSZ4oYC&index=5 |
| | Lesson 6 | Order, measure, and compare the length of objects before and after measuring with centimeter cubes, solving compare with difference unknown word problems. https://www.youtube.com/watch?v=Rv8zVuajJFY&list=PLvolZqLMhJmk7mcZ7SxjWhYG1YlSZ4oYC&index=6 |
| **Topic C:** Non-Standard and Standard Length Units | Lesson 7 | Measure the same objects from Topic B with different non-standard units simultaneously to see the need to measure with a consistent unit. https://www.youtube.com/watch?v=OCI6erDDryM&list=PLvolZqLMhJmk7mcZ7SxjWhYG1YlSZ4oYC&index=7 |
| | Lesson 8 | Understand the need to use the same units when comparing measurements with others https://www.youtube.com/watch?v=07KPutlHkYw&list=PLvolZqLMhJmk7mcZ7SxjWhYG1YlSZ4oYC&index=8 |

| | Lesson 9 | Answer compare with difference unknown problems about lengths of two different objects measured in centimeters<br><br>https://www.youtube.com/watch?v=iyagzy05G8c&index=9&list=PLvolZqLMhJmk7mcZ7SxjWhYG1YlSZ4oYC |
|---|---|---|
| **End-of- Module Assessment Task**<br>**(Interview Style: 3 days)** | | |

# NJSLS Standards:

<table>
<tr><td colspan="2" style="background-color:yellow"><strong><em>Module 3: Ordering and Comparing Length Measurements as Numbers</em></strong></td></tr>
<tr><td>1.OA.1</td><td>Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.</td></tr>
</table>

Introduce situations for students to model- starting with concrete materials; and as students are ready- working with drawings, part-part- whole representations, and number lines.

Have a variety of materials for students to model the problems. This is the true purpose of this standard. After representing the problem, teachers guide discussion on how to represent what the students modeled/drew with mathematics (equations). This follows the concrete/pictorial/abstract progression that promotes long-term conceptual understanding.

First grade students extend their experiences in Kindergarten by working with numbers to 20 to solve a new type of problem situation: Compare (See **Table 1** in this document for examples of all problem types). In a Compare situation, two amounts are compared to find "How many more" or "How many less".

As students show proficiency with models and drawings, begin to demonstrate how to represent the actions using equations. Give students the opportunity to say the equation orally and then in writing.

The vocabulary of comparison situations can cause confusion for students. While the words *more than* implies addition and *fewer than* implies subtraction, that is not always the case. Avoid teaching "Key Words"

| Problem Type: Compare | | |
|---|---|---|
| *Difference* Unknown: <br> *"How many more?" version.* <br> Lucy has 7 apples.  Julie as 9 apples.  How many more apples does Julie have than Lucy? | *Bigger* Unknown: <br> *"More" version suggests operation.* <br> Julie has 2 more apples than Lucy.  Lucy has 7 apples.  How many apples does Julie have? | Smaller Unknown: <br> *Version with "more"* <br><br> Mastery expected in Second Grade |
| *"How many fewer?" version* <br> Lucy as 7 apples.  Julie has 9 apples.  How many fewer apples does Lucy have than Julie? <br> $7 + \square = 9$ <br><br> $9 - 7 = \square$ | Bigger Unknown: <br> *Version with "fewer"* <br><br> Mastery expected in Second Grade | Smaller Unknown: <br> *"Fewer" version suggests operation.* <br> Lucy has 2 fewer apples than Julie.  Julie has 9 apples.  How many apples does Lucy have? |

| 1.MD.1 | Order three objects by length; compare the lengths of two objects indirectly by using a third object. |
|---|---|

Teach students to first predict by estimating how long or how tall an object will be
Students continue to use direction comparison to compare lengths using non-standard units.
Direct comparison means that students compare the amount of an attribute in two objects without using formal measurement units like paper clips, cubes, string and similar items.
**Example:** Who is taller?
Student 1 Let's stand back to back and compare our heights. Look! I'm taller!

In Grade 1, measurement experiences are extended to include direct comparisons-using a third object as a comparison tool for length (transitivity)
**Example:** If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C.

Provide problem-solving experiences for students to share their thinking and reasoning ordering a set of objects by length.
**Example**: The snake handler is trying to put the snakes in order from shortest to longest. Here are the three snakes (3 strings of different length and color) What order should she put the snakes?
Student: Ok. I will lay the snakes next to each other. I need to make sure to be careful and line them up so they all start at the same place. So, the blue snake is the shortest. The green snake is the longest. And the red snake is the medium-sized. So, I'll put them in order from shortest to longest: blue, red, green

| 1.MD.2 | Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measure is spanned by a whole number of length units with no gaps or overlaps. |
|---|---|

Students gain their first experience with measuring length as the iteration of a smaller, uniform length called a length unit.

Students learn that measuring the length of an object in this way requires placing length units (manipulatives of the same size) end to end without gaps or overlaps, and then counting the number of units to determine the length.

The Geometric Measurement Progression recommends beginning with actual standard

units (e.g., 1-inch cubes or centimeter cubes, referred to as length units) to measure length.

This standard limit measurement to whole numbers of length, though not all objects will measure to an exact whole unit. Students will need to adjust their answers because of this. **Example:** if a pencil actually measures between 6 and 7centimeter cubes long, the students could state the pencil is "about [6 or 7] centimeter cubes long"; they would choose the closer of the two numbers.

| 1.MD.4 | Organize, represent and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category and how many more or less are in one category than in another |
|---|---|

Students can use graphs and charts to organize and represent data about things in their lives (e.g., favorite colors, pets, shoe types, and so on).

Charts may be constructed by groups of students as well as by individual students. These activities will help prepare students for work in grade two when they draw picture graphs and bar graph.

Ensure students limit categories to only three choices

When students collect, represent, and interpret data, they reinforce number sense and counting skills. When students ask and answer questions about information in charts or graphs, they sort and compare data. Students use addition and subtraction and comparative language and symbols to interpret graphs and charts

Require students to write 2 sentences about their graphs including analysis of the most and least in a category and a third sentence to tell how many more or less are in one category than another.
This standard is designed to work well with the compare situations (see Table 1)
Data can be collected and organized, graphed, and/or displayed throughout daily activities such as calendar, weather, attendance, question of the day.

M: Major Content          S: Supporting Content          A: Additional Content

Common addition and subtraction. [1]

|  | RESULT UNKNOWN | CHANGE UNKNOWN | START UNKNOWN |
|---|---|---|---|
| ADD TO | Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$ | Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$ | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$ |
| TAKE FROM | Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$ | Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$ | Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$ |
|  | TOTAL UNKNOWN | ADDEND UNKNOWN | BOTH ADDENDS UNKNOWN [2] |
| PUT TOGETHER / TAKE APART [3] | Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$ | Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$ | Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? $5 = 0 + 5, 5 + 0$ $5 = 1 + 4, 5 = 4 + 1, 5 = 2 + 3, 5 = 3 + 2$ |
| COMPARE | DIFFERENCE UKNOWN | BIGGER UNKNOWN | SMALLER UNKNOWN |
|  | ("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? $2 + ? = 5, 5 - 2 = ?$ | (Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$ | (Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, ? + 3 = 5$ |

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

**http://www.corestandards.org/Math/Content/mathematics-glossary/Table-1/**

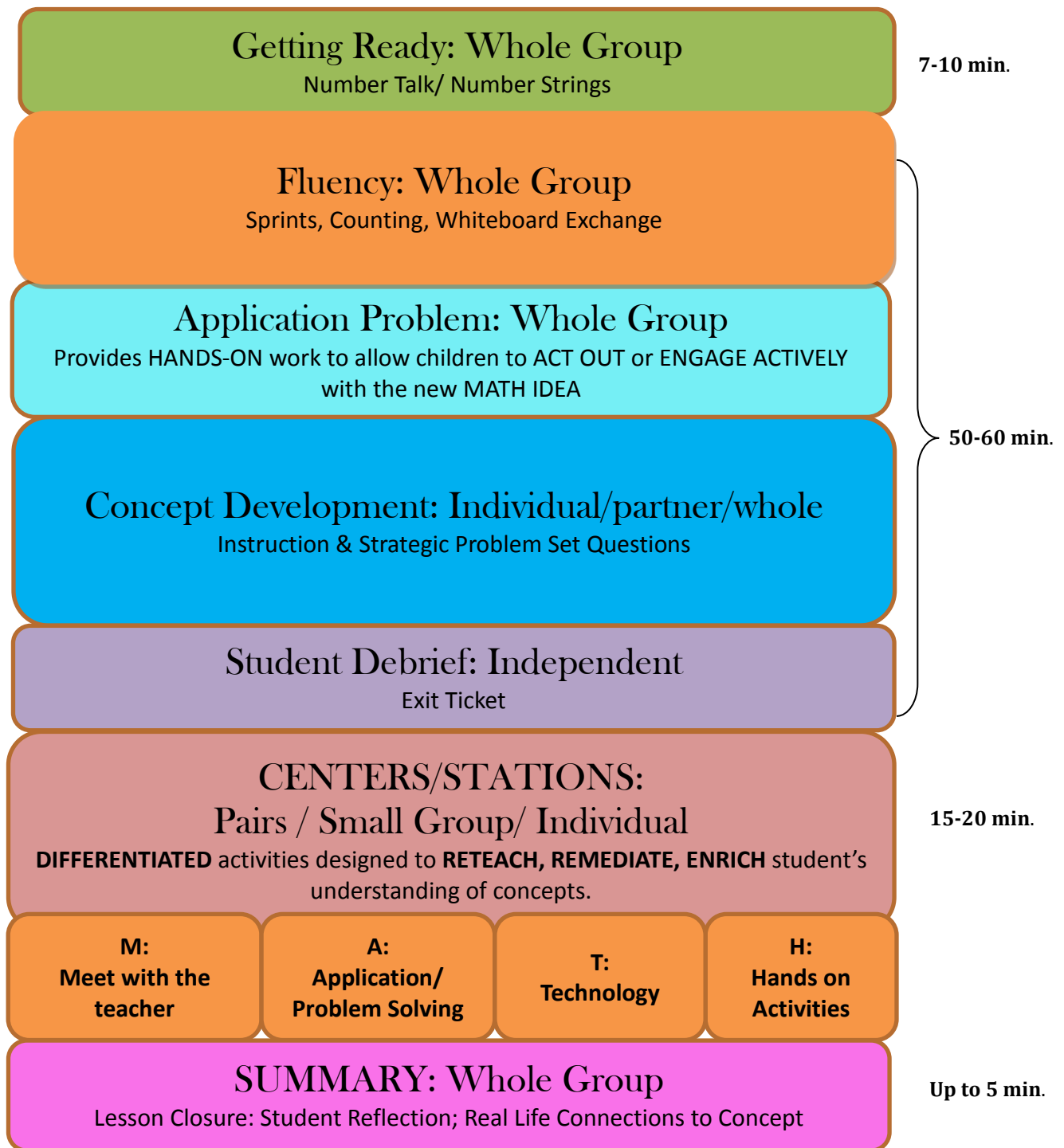| Teaching Representations/ Manipulatives/ Tools: |
| --- |

- Centimeter cubes

- Centimeter rulers (simply for the purpose of naming the centimeter)

- Non-standard units (toothpicks, small and large paper clips)

- String lengths of about 25 centimeters

- Tally mark

## Terminology/ Symbols

| | |
| --- | --- |
| - Centimeter (standard length unit within the metric system)<br>- Centimeter cube (pictured to the right, also used as a length unit in this module)<br>- Centimeter ruler (measurement tool using length units of centimeters)<br>- Data (collected information)<br>- Endpoint (the end of an object, referenced when aligning for measurement purposes) | - Length unit (measuring the length of an object with equal-sized units)<br>- Poll (survey)<br>- Table or graph (organized charts visually representing data)<br>- Height (measurement of vertical distance of an object) |

| Module 3 Assessment / Authentic Assessment Recommended Framework | | | |
|---|---|---|---|
| Assessment | CCSS | Estimated Time | Format |
| *Eureka Math* <br> *Module 3: Ordering and Comparing Length Measurements as Numbers* | | | |
| Portfolio/Authentic Assessment: Measuring with Paperclips | 1.MD.2 | 30 mins | Individual |
| Optional End of Module Assessment | 1.OA.1 <br> 1.MD.1 <br> 1.MD.2 <br> 1.MD.4 | 1 Block | Individual |

# First Grade Ideal Math Block

**Getting Ready: Whole Group**
Number Talk/ Number Strings

**7-10 min.**

**Fluency: Whole Group**
Sprints, Counting, Whiteboard Exchange

**Application Problem: Whole Group**
Provides HANDS-ON work to allow children to ACT OUT or ENGAGE ACTIVELY with the new MATH IDEA

**Concept Development: Individual/partner/whole**
Instruction & Strategic Problem Set Questions

**Student Debrief: Independent**
Exit Ticket

**50-60 min.**

**CENTERS/STATIONS:**
Pairs / Small Group/ Individual
DIFFERENTIATED activities designed to RETEACH, REMEDIATE, ENRICH student's understanding of concepts.

**15-20 min.**

| M: Meet with the teacher | A: Application/ Problem Solving | T: Technology | H: Hands on Activities |
|---|---|---|---|

**SUMMARY: Whole Group**
Lesson Closure: Student Reflection; Real Life Connections to Concept

**Up to 5 min.**

| Eureka Lesson Structure: |
|:---:|

**Fluency:**

- Sprints

- Counting: Can start at numbers other than 0 or 1 and might include supportive concrete material or visual models

- Whiteboard Exchange

**Application Problem:**

- Engage students in using the RDW Process

- Sequence problems from simple to complex and adjust based on students' responses

- Facilitate share and critique of various explanations, representations, and/or examples.

**Concept Development:** (largest chunk of time)

Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.

- Use of tools, precise mathematical language, and/or models

- Balance teacher talk with opportunities for peer share and/or collaboration

- Generate next steps by watching and listening for understanding

Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle

- Assign problems strategically to differentiate practice as needed

- Create and assign remedial sequences as needed

**Student Debrief:**

- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion

- Culminate with students' verbal articulation of their learning for the day

- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

| PARCC Assessment Evidence/Clarification Statements | | | |
|---|---|---|---|
| **CCSS** | **Evidence Statement** | **Clarification** | **Math Practices** |
| 1.OA.A.1 | Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknown in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. | i) Tasks should include all problem situations and all of their subtypes and language variants. Mastery is expected in "Add To" and "Take From" - Result and Change Unknown Problems, "Put Together/Take Apart" Problems, "Compare" – Difference Unknown, Bigger Unknown (more version) and Smaller Unknown (fewer version) Problems (for more information see CCSS Table 1 and OA Progression, p. 9.)<br>ii) Interviews (individual or small group) are used to assess mastery of different problem types. | MP 1, 4 |

# Number Talks

**What does Number Talks look like?**

- Students are near each other so they can communicate with each other (central meeting place)
- Students are mentally solving problems
- Students are given thinking time
- Thumbs up show when they are ready
- Teacher is recording students' thinking

**Communication**

- Having to talk out loud about a problem helps students clarify their own thinking
- Allow students to listen to other's strategies and value other's thinking
- Gives the teacher the opportunity to hear student's thinking

**Mental Math**

- When you are solving a problem mentally you must rely on what you know and understand about the numbers instead of memorized procedures
- You must be efficient when computing mentally because you can hold a lot of quantities in your head

**Thumbs Up**

- This is just a signal to let you know that you have given your students enough time to think about the problem
- If will give you a picture of who is able to compute mentally and who is struggling
- It isn't as distracting as a waving hand

**Teacher as Recorder**

- Allows you to record students' thinking in the correct notation
- Provides a visual to look at and refer back to
- Allows you to keep a record of the problems posed and which students offered specific strategies

**Purposeful Problems**

- Start with small numbers so the students can learn to focus on the strategies instead of getting lost in the numbers
- Use a number string (a string of problems that are related to and scaffold each other)

**Starting Number Talks in your Classroom**

- Start with specific problems in mind
- Be prepared to offer a strategy from a previous student
- It is ok to put a student's strategy on the backburner
- Limit your number talks to about 15 minutes
- Ask a question, don't tell!

**The teacher asks questions:**

- Who would like to share their thinking?
- Who did it another way?
- How many people solved it the same way as Billy?
- Does anyone have any questions for Billy?
- Billy, can you tell us where you got that 5?
- How did you figure that out?
- What was the first thing your eyes saw, or your brain did?

Student Name: _____ Task: _____ School: _____ Teacher: _____ Date: _____

| "I CAN…" | STUDENT FRIENDLY RUBRIC | | | | SCORE |
|---|---|---|---|---|---|
| | …a start 1 | …getting there 2 | …that's it 3 | WOW! 4 | |
| Understand | I need help. | I need some help. | I do not need help. | I can help a class-mate. | |
| Solve | I am unable to use a strategy. | I can start to use a strategy. | I can solve it more than one way. | I can use more than one strategy and talk about how they get to the same answer. | |
| Say or Write | I am unable to say or write. | I can write or say some of what I did. | I can write and talk about what I did.<br><br>I can write or talk about why I did it. | I can write and say what I did and why I did it. | |
| Draw or Show | I am not able to draw or show my thinking. | I can draw, but not show my thinking; or I can show but not draw my thinking; | I can draw and show my thinking | I can draw, show and talk about my think-ing. | |

# Use and Connection of Mathematical Representations
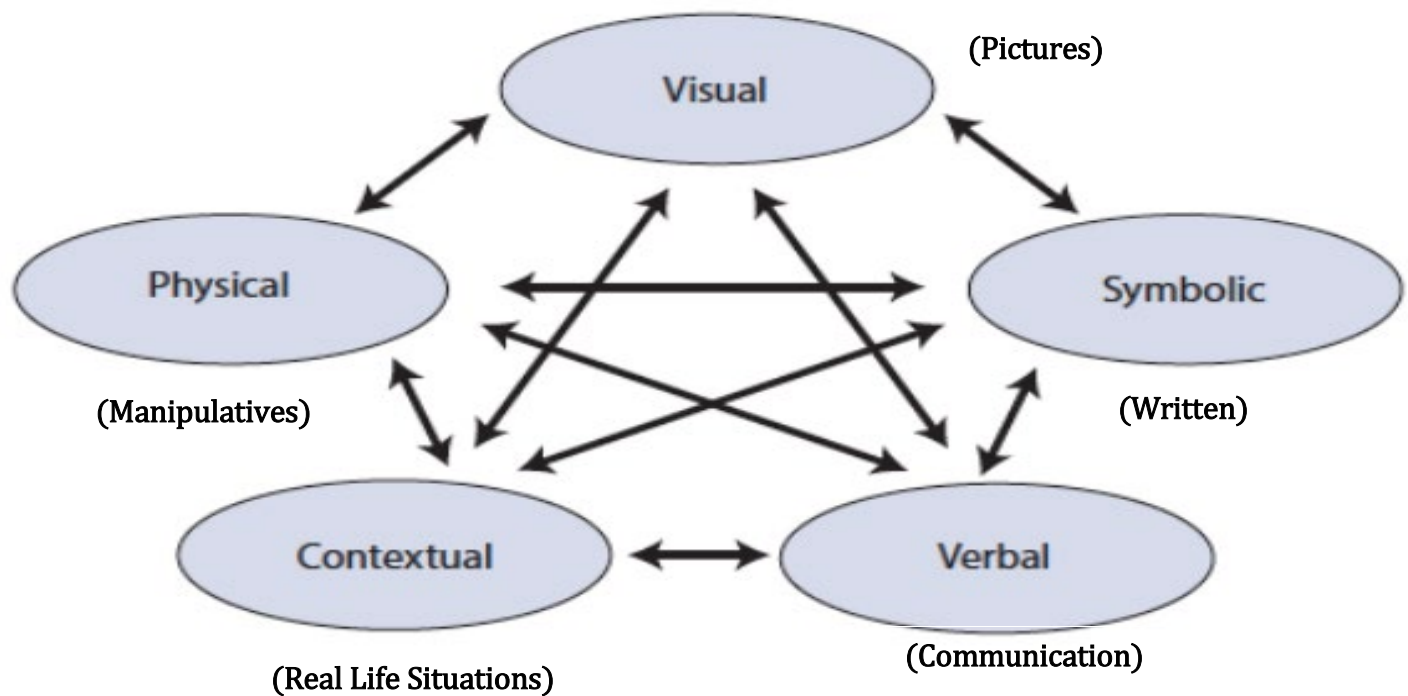


The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

**Visual:** When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

**Physical**: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

**Verbal**: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

**Symbolic**: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

**Contextual:** A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

### The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

## Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

> **Concrete:** "Doing Stage": Physical manipulation of objects to solve math problems.
>
> **Pictorial:** "Seeing Stage": Use of imaged to represent objects when solving math problems.
>
> **Abstract:** "Symbolic Stage": Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

---

## Read, Draw, Write Process

**READ** the problem. Read it over and over…. And then read it again.

**DRAW** a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

**WRITE** your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

# Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

Teacher Questioning:

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple "yes" or "no," or do they invite students to deepen their understanding?



The most important thing is to NEVER stop questioning

Albert Einstein

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

## Help students learn to reason mathematically

**26** How did you **begin** to think about this problem?

**27** What is **another way** you could solve this problem?

**28** How could you **prove** _____?

**29** Can you **explain how your answer is different from or the same as** [student]'s answer?

**30** Let's **break the problem into parts**. What would the parts be?

**31** Can you **explain this part more specifically**?

**32** Does that **always work**?

**33** Can you think of a case where that **wouldn't work**?

**34** How did you **organize** your information? Your thinking?

## Help students evaluate their own processes and engage in productive peer interaction

**35** What do you need to do **next**?

**36** What have you **accomplished**?

**37** What are your **strengths and weaknesses**?

**38** Was your **group participation appropriate and helpful**?

## Help students with problem comprehension

**39** What is this problem about? What can you **tell me about it**?

**40** Do you need to **define or set limits** for the problem?

**41** How would you **interpret** that?

**42** Could you **reword that in simpler terms**?

**43** Is there something that can be **eliminated** or that is **missing**?

**44** Could you **explain** what the problem is asking?

**45** What **assumptions** do you have to make?

**46** What do you **know** about this part?

**47** Which words were **most important**? Why?

# Help students learn to **conjecture, invent, and solve** problems

48 **What would happen if ___?**

49 Do you see a **pattern**?

50 What are some **possibilities** here?

51 Where could you find the **information** you need?

52 How would you **check your steps** or your answer?

53 What **did not work**?

54 How is your solution method the **same as or different from** [student]'s method?

55 Other than retracing your steps, **how can you determine** if your answers are appropriate?

56 How did you **organize** the information? Do you have a **record**?

57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?

58 What have you tried? What **steps** did you take?

59 How would it look if you used this **model** or these **materials**?

60 How would you draw a **diagram or make a sketch** to solve the problem?

61 Is there **another possible answer**? If so, explain.

62 Is there **another way to solve** the problem?

63 Is there **another model** you could use to solve the problem?

64 Is there anything you've **overlooked**?

65 **How did you think** about the problem?

66 What was your **estimate or prediction**?

67 How **confident** are you in your answer?

68 **What else** would you like to know?

69 What do you think comes **next**?

70 Is the solution **reasonable**, considering the context?

71 Did you have a **system**? Explain it.

72 Did you have a **strategy**? Explain it.

73 Did you have a **design**? Explain it.

# Help students learn to connect mathematics, its ideas, and its application

74. What is the **relationship** between ___ and ___?

75. Have we ever solved a problem **like this before**?

76. What uses of mathematics did you find in the **newspaper** last night?

77. What is the **same**?

78. What is **different**?

79. Did you use skills or build on concepts that were **not necessarily mathematical**?

80. Which **skills or concepts** did you use?

81. What **ideas** have we explored before that were useful in solving this problem?

82. Is there a **pattern**?

83. **Where else** would this strategy be useful?

84. How does this **relate** to ___?

85. Is there a **general rule**?

86. Is there a **real-life situation** where this could be used?

87. How would your method work with **other problems**?

88. What other problem does this seem to **lead to**?

## Help students persevere

89. Have you tried making a **guess**?

90. **What else** have you tried?

91. Would **another method** work as well or better?

92. Is there **another way** to draw, explain, or say that?

93. Give me another **related problem**. Is there an easier problem?

94. How would you **explain** what you know right now?

95. What was **one thing you learned** (or two, or more)?

96. Did you **notice any patterns**? If so, describe them.

97. What **mathematics topics** were used in this investigation?

98. What were the **mathematical ideas** in this problem?

99. What is mathematically **different about these two situations**?

100. What are the **variables** in this problem? What stays **constant**?

## Help students focus on the mathematics from activities

# Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

# Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

# Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

### K-2 Math Fact Fluency Expectation

**K.OA.5** Add and Subtract within 5.
**1.OA.6** Add and Subtract within 10.
**2.OA.2** Add and Subtract within 20.

## Math Fact Fluency: Fluent Use of Mathematical Strategies

First and second grade students are expected to solve addition and subtraction facts using a variety of strategies fluently.

**1.OA.6**    Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Use strategies such as:
- counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14);
- decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9);
- using the relationship between addition and subtraction; and
- creating equivalent but easier or known sums.

**2.NBT.7** Add and subtract within 1000, using concrete models or drawings and strategies based on:
- place value,
- properties of operations, and/or
- the relationship between addition and subtraction;

# Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

## Mathematical Proficiency

To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;

- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;

- Strategic competence: ability to formulate, represent, and solve mathematical problems;

- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;

- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

## Evidence should:

- Provide a window in student thinking;

- Help teachers to determine the extent to which students are reaching the math learning goals; and

- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.
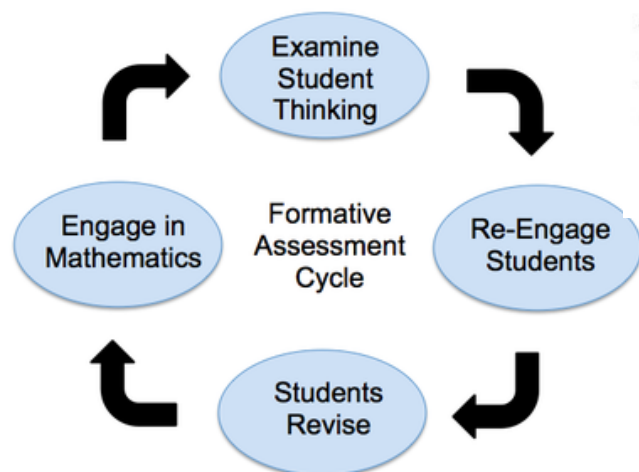


*Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.*

(Wiliam 2007, pp. 1054; 1091)

# Connections to the Mathematical Practices

## Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

| | |
|---|---|
| The **Standards for Mathematical Practice** describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. | |
| | **Make sense of problems and persevere in solving them** |
| 1 | Mathematically proficient students in First Grade continue to develop the ability to focus attention, test hypotheses, take reasonable risks, remain flexible, try alternatives, exhibit self-regulation, and persevere (Copley, 2010). As the teacher uses thoughtful questioning and provides opportunities for students to share thinking, First Grade students become conscious of what they know and how they solve problems. They make sense of task-type problems, find an entry point or a way to begin the task, and are willing to try other approaches when solving the task. They ask themselves, "Does this make sense?" First Grade students' conceptual understanding builds from their experiences in Kindergarten as they continue to rely on concrete manipulatives and pictorial representations to solve a problem, eventually becoming fluent and flexible with mental math as a result of these experiences. |
| 2 | Reason abstractly and quantitatively |

Mathematically proficient students in First Grade recognize that a number represents a specific    quantity. They use numbers and symbols to represent a problem, explain thinking, and justify a        response. For example, when solving the problem: "There are 60 children on the playground. Some children line up. There are 20 children still on the playground. How many children lined up?" first grade students may write 20 + 40 = 60 to indicate a Think-Addition strategy. Other students may illustrate a counting-on by tens strategy by writing 20 + 10 + 10 + 10 + 10 = 60. The numbers and equations written illustrate the students' thinking and the strategies used, rather than how to simply compute, and how the story is decontextualized as it is represented abstractly with symbols.

| | |
|---|---|
| | **Construct viable arguments and critique the reasoning of others** |
| 3 | Mathematically proficient students in First Grade continue to develop their ability to clearly express, explain, organize and consolidate their math thinking using both verbal and written representations. Their understanding of grade appropriate vocabulary helps them to construct viable arguments about mathematics. For example, when justifying why a particular shape isn't a square, a first grade student may hold up a picture of a rectangle, pointing to the various parts, and reason, "It can't be a square because, even though it has 4 sides and 4 angles, the sides aren't all the same size." In a classroom where risk-taking and varying perspectives are encouraged, mathematically proficient students are willing and eager to share their ideas with others, consider other ideas proposed by classmates, and question ideas that don't seem to make sense. |
| | **Model with mathematics** |
| 4 | Mathematically proficient students in First Grade model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. They also use tools, such as tables, to help collect information, analyze results, make conclusions, and review their conclusions to see if the results make sense and revising as needed. |

| | |
|---|---|
| | **Use appropriate tools strategically** |
| 5 | Mathematically proficient students in First Grade have access to a variety of concrete (e.g. 3-dimensional solids, ten frames, number balances, number lines) and technological tools (e.g., virtual manipulatives, calculators, interactive websites) and use them to investigate mathematical concepts. They select tools that help them solve and/or illustrate solutions to a problem. They recognize that multiple tools can be used for the same problem- depending on the strategy used. For example, a child who is in the counting stage may choose connecting cubes to solve a problem. While, a student who understands parts of number, may solve the same problem using ten-frames to decompose numbers rather than using individual connecting cubes. As the teacher provides numerous opportunities for students to use educational materials, first grade students' conceptual understanding and higher order thinking skills are developed |
| | **Attend to precision** |
| 6 | Mathematically proficient students in First Grade attend to precision in their communication, calculations, and measurements. They are able to describe their actions and strategies clearly, using grade-level appropriate vocabulary accurately. Their explanations and reasoning regarding their process of finding a solution becomes more precise. In varying types of mathematical tasks, first grade students pay attention to details as they work. For example, as students' ability to attend to position and direction develops, they begin to notice reversals of numerals and self-correct when appropriate. When measuring an object, students check to make sure that there are not any gaps or overlaps as they carefully place each unit end to end to measure the object (iterating length units). Mathematically proficient first grade students understand the symbols they use (=, >, 3, a |

| | |
|---|---|
| | proficient student who is able to attend to precision states, "Four is more than 3" rather than "The alligator eats the four. It's bigger." |
| | **Look for and make use of structure** |
| 7 | Mathematically proficient students in First Grade carefully look for patterns and structures in the number system and other areas of mathematics. For example, while solving addition problems using a number balance, students recognize that regardless whether you put the 7 on a peg first and then the 4, or the 4 on first and then the 7, they both equal 11 (commutative property). When decomposing two-digit numbers, students realize that the number of tens they have constructed 'happens' to coincide with the digit in the tens place. When exploring geometric properties, first graders recognize that certain attributes are critical (number of sides, angles), while other properties are not (size, color, orientation) |
| | **Look for and express regularity in repeated reasoning** |
| 8 | Mathematically proficient students in First Grade begin to look for regularity in problem structures when solving mathematical tasks. For example, when adding three one-digit numbers and by making tens or using doubles, students engage in future tasks looking for opportunities to employ those same strategies. Thus, when solving 8+7+2, a student may say, "I know that 8 and 2 equal 10 and then I add 7 more. That makes 17. It helps to see if I can make a 10 out of 2 numbers when I start." Further, students use repeated reasoning while solving a task with multiple correct answers. For example, in the task "There are 12 crayons in the box. Some are red, and some are blue. How many of each could there be?" First Grade students realize that the 12 crayons could include 6 of each color (6+6 = 12), 7 of one color and 5 of another (7+5 = 12), etc. In essence, students repeatedly find numbers that add up to 12. |

## Effective Mathematics Teaching Practices

**Establish mathematics goals to focus learning**. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

**Implement tasks that promote reasoning and problem solving**. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

**Use and connect mathematical representations**. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

**Facilitate meaningful mathematical discourse**. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

**Pose purposeful questions**. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

**Build procedural fluency from conceptual understanding**. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

**Support productive struggle in learning mathematics**. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

**Elicit and use evidence of student thinking**. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

| **5 Practices for Orchestrating Productive Mathematics Discussions** | |
| --- | --- |
| Practice | Description/ Questions |
| 1. Anticipating | What strategies are students likely to use to approach or solve a challenging high-level mathematical task?<br><br>How do you respond to the work that students are likely to produce?<br><br>Which strategies from student work will be most useful in addressing the mathematical goals? |
| 2. Monitoring | Paying attention to what and how students are thinking during the lesson.<br><br>Students working in pairs or groups<br><br>Listening to and making note of what students are discussing and the strategies they are using<br><br>Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle) |
| 3. Selecting | This is the process of deciding the *what* and the *who* to focus on during the discussion. |
| 4. Sequencing | What order will the solutions be shared with the class? |
| 5. Connecting | Asking the questions that will make the mathematics explicit and understandable.<br><br>Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations. |

# MATH CENTERS/ WORKSTATIONS

*Math workstations* allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

## Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills.  Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated**. If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.



In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.
In her book, she leads you step-by-step through the process of implementing work stations.

# MATH WORKSTATION INFORMATION CARD

Math Workstation: _____     Time: _____

NJSLS.:

_____
_____
_____
_____

Objective(s): By the end of this task, I will be able to:

- _____
- _____
- _____

Task(s):

- _____
- _____
- _____
- _____

Exit Ticket:

- _____
- _____
- _____

## MATH WORKSTATION SCHEDULE                                          Week of: _____

| DAY | Technology Lab | Problem Solving Lab | Fluency Lab | Math Journal | Small Group Instruction |
|---|---|---|---|---|---|
| Mon. | Group ____ | Group ____ | Group ____ | Group ____ | BASED ON CURRENT OBSERVATIONAL DATA |
| Tues. | Group ____ | Group ____ | Group ____ | Group ____ | |
| Wed. | Group ____ | Group ____ | Group ____ | Group ____ | |
| Thurs. | Group ____ | Group ____ | Group ____ | Group ____ | |
| Fri. | Group ____ | Group ____ | Group ____ | Group ____ | |

## INSTRUCTIONAL GROUPING

| | GROUP A | | GROUP B |
|---|---|---|---|
| 1 | | 1 | |
| 2 | | 2 | |
| 3 | | 3 | |
| 4 | | 4 | |
| 5 | | 5 | |
| 6 | | 6 | |

| | GROUP C | | GROUP D |
|---|---|---|---|
| 1 | | 1 | |
| 2 | | 2 | |
| 3 | | 3 | |
| 4 | | 4 | |
| 5 | | 5 | |
| 6 | | 6 | |

# First Grade PLD Rubric

| Got It | | Not There Yet | | |
|---|---|---|---|---|
| Evidence shows that the student essentially has the target concept or big math idea. | | Student shows evidence of a major misunderstanding, incorrect concepts or procedure, or a failure to engage in the task. | | |
| PLD Level 5: 100% Distinguished command | PLD Level 4: 89% Strong Command | PLD Level 3: 79% Moderate Command | PLD Level 2: 69% Partial Command | PLD Level 1: 59% Little Command |
| Student work shows **distinguished levels of understanding** of the mathematics.<br><br>Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the:<br>• Tools:<br>  o Manipulatives<br>  o Five Frame<br>  o Ten Frame<br>  o Number Line<br>  o Part-Part-Whole Model<br>• Strategies:<br>  o Drawings<br>  o Counting All<br>  o Count On/Back<br>  o Skip Counting<br>  o Making Ten<br>  o Decomposing Number<br>• Precise use of math vocabulary<br>Response includes an **efficient and logical progression** of mathematical reasoning and understanding. | Student work shows **strong levels of understanding** of the mathematics.<br><br>Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the:<br>• Tools:<br>  o Manipulatives<br>  o Five Frame<br>  o Ten Frame<br>  o Number Line<br>  o Part-Part-Whole Model<br>• Strategies:<br>  o Drawings<br>  o Counting All<br>  o Count On/Back<br>  o Skip Counting<br>  o Making Ten<br>  o Decomposing Number<br>• Precise use of math vocabulary<br><br>Response includes a **logical progression** of mathematical reasoning and understanding. | Student work shows **moderate levels of understanding** of the mathematics.<br><br>Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the:<br>• Tools:<br>  o Manipulatives<br>  o Five Frame<br>  o Ten Frame<br>  o Number Line<br>  o Part-Part-Whole Model<br>• Strategies:<br>  o Drawings<br>  o Counting All<br>  o Count On/Back<br>  o Skip Counting<br>  o Making Ten<br>  o Decomposing Number<br>• Precise use of math vocabulary<br><br>Response includes a **logical but incomplete progression** of mathematical reasoning and understanding.<br>Contains **minor errors**. | Student work shows **partial understanding** of the mathematics.<br><br>Student **constructs** and **communicates** an **incomplete response** based on student's attempts of explanations/ reasoning using the:<br>• Tools:<br>  o Manipulatives<br>  o Five Frame<br>  o Ten Frame<br>  o Number Line<br>  o Part-Part-Whole Model<br>• Strategies:<br>  o Drawings<br>  o Counting All<br>  o Count On/Back<br>  o Skip Counting<br>  o Making Ten<br>  o Decomposing Number<br>• Precise use of math vocabulary<br><br>Response includes an **incomplete or illogical progression** of mathematical reasoning and understanding. | Student work shows **little understanding** of the mathematics.<br><br>Student **attempts** to **constructs** and **communicates** a response using the:<br>• Tools:<br>  o Manipulatives<br>  o Five Frame<br>  o Ten Frame<br>  o Number Line<br>  o Part-Part-Whole Model<br>• Strategies:<br>  o Drawings<br>  o Counting All<br>  o Count On/Back<br>  o Skip Counting<br>  o Making Ten<br>  o Decomposing Number<br>• Precise use of math vocabulary<br><br>Response includes **limited evidence of the progression** of mathematical reasoning and understanding. |
| 5 points | 4 points | 3 points | 2 points | 1 point |

# DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

Answering these questions will help inform instructional decisions and will influence lesson planning.

Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?

Now it is time to begin the analysis again.
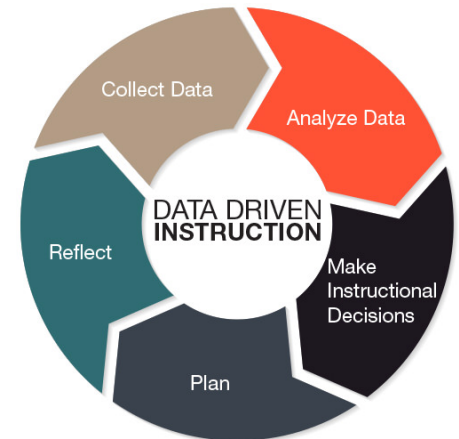
# Data Analysis Form     School: _____     Teacher: _____     Date: _____
## Assessment: _____     NJSLS: _____

| GROUPS (STUDENT INITIALS) | SUPPORT PLAN | PROGRESS |
|---|---|---|
| MASTERED (86% - 100%) (PLD 4/5): | | |
| DEVELOPING (67% - 85%) (PLD 3): | | |
| INSECURE (51%-65%) (PLD 2): | | |
| BEGINNING (0%-50%) (PLD 1): | | |

\

## MATH PORTFOLIO EXPECTATIONS

**The Student Assessment Portfolios for Mathematics** are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the CCSS-M. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSLS and be "practice forward" (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

## K-2 GENERAL PORTFOLIO EXPECTATIONS:

- Tasks contained within the Student Assessment Portfolios are "practice forward" and denoted as "Individual", "Partner/Group", and "Individual w/Opportunity for Student Interviews[1].
- Each Student Assessment Portfolio should contain a "Task Log" that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity "as a new and separate score" in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is *recommended.*
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)[2].
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

## GRADES K-2

### Student Portfolio Review

Provide students the opportunity to review and evaluate their portfolio at various points throughout the year; celebrating their progress and possibly setting goals for future growth. During this process, students should retain ALL of their current artifacts in their Mathematics Portfolio

## Measuring with Paper Clips
Performance Task

Name: _____     Teacher: _____     Date: _____

Joe measured the length of the line using paper clips. He told his teacher that it took 8 paper clips. Do you think Joe is correct? Why? Show your thinking with pictures, numbers, or words.

| Measuring with Paper Clips | |
|---|---|
| **Domain** | Measurement and Data |
| **Cluster** | Measure lengths indirectly and by iterating length units. |
| **Standard(s)** | **1.MD.2** Express the length of an object as a whole number of length units, by laying multiple copies of a short object end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. |
| **Materials** | SF of line measured incorrectly by paperclips, pencil |
| **Task** | Provide materials to the student. Read the problem to the student: *Joe measured the length of the line using paper clips. He told his teacher that it took 8 paperclips. Do you think Joe is correct? Why? Why Not? Show your thinking with pictures, numbers, or words.* |

| Continuum of Understanding | |
|---|---|
| **Developing Understanding** | • Counts 8 paperclips and states that Joe is correct.<br>• Does not notice that the paperclips are overlapping and have gaps. |
| **Complete Understanding** | • Correctly identifies that Joe measured the length of the line incorrectly.<br>• States the importance of carefully measuring an object without any overlaps or gaps. |

| Standards for Mathematical Practice |
|---|
| **1. Makes sense and perseveres in solving problems.** |
| 2. Reasons abstractly and quantitatively. |
| **3. Constructs viable arguments and critiques the reasoning of others.** |
| 4. Models with mathematics. |
| 5. Uses appropriate tools strategically. |
| **6. Attends to precision.** |
| 7. Looks for and makes use of structure. |
| 8. Looks for and expresses regularity in repeated reasoning. |

| Level 5: Distinguished Command | Level 4: Strong Command | Level 3: Moderate Command | Level 2: Partial Command | Level 1: No Command |
|---|---|---|---|---|
| Student correctly answers and clearly constructs and communicates a complete response based on explanations/reasoning using:<br><br>• Measurement as the covering of distance (no gaps or overlaps in the placement of units)<br><br>Response includes an **efficient** and logical progression of steps.<br><br>Strategy and execution meet the content, process, and qualitative demands of the task or concept. Student can communicate ideas. | Student correctly answers and clearly constructs and communicates a complete response with one minor error based on explanations/reasoning using:<br><br>• Measurement as the covering of distance (no gaps or overlaps in the placement of units)<br><br>Response includes a **logical** progression of steps<br><br>May have minor errors that do not impact the mathematics. | Student answers, clearly constructs, and communicates a complete response with minor errors based on explanations/reasoning using:<br><br>• Measurement as the covering of distance (no gaps or overlaps in the placement of units)<br><br>Response includes a **logical but incomplete** progression of steps. Minor calculation errors. | The task is attempted, and some mathematical effort is made. There may be fragments of accomplishment but little success.<br><br>• Measurement as the covering of distance (no gaps or overlaps in the placement of units)<br><br>Response includes an **incomplete or Illogical** progression of steps. | The student shows no work or justification. Further teaching is required. |

# Resources

**Number Book Assessment** Link: **http://investigations.terc.edu/**

**Model Curriculum**- http://www.nj.gov/education/modelcurriculum/

**Georgia Department of Education: Games to be played at centers with a partner or small group.** http://ccgpsmathematicsk-5.wikispaces.com/Kindergarten

**Engage NY: *For additional resources to be used during centers or homework.**
https://www.engageny.org/sites/default/files/resource/attachments/math-gk-m1-full-module.pdf

**Add/ Subtract Situation Types:**   Darker Shading indicates Kindergarten expectations
https://achievethecore.org/content/upload/Add%20Subtract%20Situation%20Types.pdf

**Math in Focus PD Videos:** https://www-k6.thinkcentral.com/content/hsp/math/hspmath/common/mif_pd_vid/9780547760346_te/index.html

# Suggested Literature

*Fish Eyes* by, Lois Ehlert

*Ten Little Puppies* by, Elena Vazquez

*Zin! Zin! Zin! A Violin!* by, Lloyd Moss

*My Granny Went to the Market* by, Stella Blackstone and Christopher Corr

*Anno's Counting Book* by, Mitsumasa Anno

*Chicka, Chicka, 1,2,3* by, Bill Martin Jr.; Michael Sampson; Lois Ehlert

*How Dinosaurs Count to 10* by Jane Yolen and Mark Teague

*10 Little Rubber Ducks* by Eric Carle

*Ten Black Dots* by Donald Crews

*Mouse Count* by Ellen Stoll Walsh

*Count!* by Denise Fleming

# 21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .

# References

"Eureka Math" *Great Minds.* 2018 < https://greatminds.org/account/products>